Mechanical Modeling for Computer Scientists
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Covered last time – Basics of mechanical systems, Free-body diagrams, equations of motion.

Modeling Methods

- Newton Euler Formulation: Sum of forces = ma.
    - D'Alembert's form: Sum of forces = 0 if the inertia of the mass is considered a force. This is essentially a simple rewriting of Newton's second law.
- Lagrangian – Difference between potential and kinetic of the system. Derives the equations of motion from the system holistically instead of looking at each mass
    - $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = F$, for each generalized coordinate x
    - Is comparable to Hamiltonian, which is not used as far as Alex knows.

Questions

- What method is the most modular?
- What are the formal definitions of free-body diagrams and system diagrams? (for computational purposes)

Analyzing Mechanical Systems

- 1st Order Systems: $m\ddot{x} + b\dot{x} = f$ (no stiffness)
    - Same form as $m\dot{x} + bx = f$ (no mass)
    - Solve with Laplace transform

      $m\dot{v} + bv = f$

      $sm(V - v_0) + bV = F$

        - $V = \frac{v_0}{s + b/m}$ Zero force input example

          $v = v_0 e^{-b/m\, t}$

          $sm(V - v_0) + bV = F$

        - $V = \frac{v_0}{s + b/m} + \frac{A/m}{s(s + b/m)}$ Constant force input example

          $v = v_0 e^{-b/m\, t} + \frac{A}{m}(1 - e^{-b/m\, t})$

- General Solutions:
  - $x = Ae^{-t/\tau}$ - Non zero initial condition
  - $x = \tau A + [x_0 - \tau A]e^{-t/\tau}$ - Input force and nonzero initial condition